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Calculation of the Replacement Time for the Fuzzy Replacement Problem using Fuzzy Ranking Method

Abstract

In almost field of our real life situations, we deal with a replacement problem, when some items such as machines, medical equipment, military tank, electric bulb etc. or workers need to replace due to their decreased efficiency, failure or break down. To have a more realistic view of a replacement problem, we consider that the capital cost ($\tilde{C}$), scrap value or resale value ($\tilde{S}$), maintenance cost or running cost ($\tilde{f}_t$) are all of fuzzy numbers. These fuzzy numbers are considered as a trapezoidal fuzzy numbers or triangular fuzzy numbers. Here we used Yager's Ranking Method to determine the best alternative of fuzzy numbers. It has been also used to transform a replacement model with fuzzy cost to a replacement model with crisp cost.

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Keywords: (C) : Capital Cost
(S) : Scrap Value
(f_t) : Maintenance / Running Cost.
TrFNs : Trapezoidal Fuzzy Numbers
TFNs : Triangular Fuzzy Numbers
RP : Replacement Problem
$\tilde{C}$, $\tilde{S}$, $\tilde{f}_t$: these are fuzzy numbers.

Introduction

Replacement Problem (RP) is one of the practical areas in economic decision analysis for our real world system. It is used in engineering economics to determine an optimal decision for maintenance and replacement purposes. When an equipment or item is used for a long time, this item tends to worsen. Then, the need for replacement becomes necessary due to a loss of efficiency. Operating equipment for a long time, the cost of operation and the maintenance cost are bound to increase year by year. A stage will come, when the maintenance cost becomes significantly large. Therefore, it is more economical to replace the equipment with new one.

Objective of the Study

In this paper, we study a realistic view that the capital cost($\tilde{C}$), scrap value ($\tilde{S}$), maintenance or running cost ($\tilde{f}_t$) of equipment are all of trapezoidal fuzzy numbers (TrFNs) or triangular fuzzy numbers (TFNs). Here, we also consider the replacement of equipment or items that deteriorate with time and the value of money does not change with time. The comparison of minimum average costs is done based on average cost. It is obtained when the total cost of the capital in owning the item and its operation cost is accumulated for n years and this total is divided by n. Here, our objective is to seek for the minimum average fuzzy cost, which cannot be obtained directly. Therefore, we need a ranking method to find out the best alternative. To deal with this situation, we use Yager's ranking method known as robust ranking method. It satisfies compensation, linearity, and additive properties.Yager's ranking method has been used to transform the fuzzy replacement problem to crisp version so that any conventional method can be applied to solve the problem. Finally, we consider only the discrete values for the cost of various times (year, hour, etc).

Preliminaries of Fuzzy Sets

Definition

Fuzzy set: A fuzzy set $\tilde{A}$ in a universe of discourse X is defined by $\tilde{A} = \{ x, \mu_{\tilde{A}}(x) \mid x \varepsilon X\}$,where $\mu_{\tilde{A}}(x) : X \rightarrow [0,1]$ is calledthe membership function of Ã and $\mu_{\tilde{A}}(x)$ is the degree of membership to which xε Ã.

Definition

A fuzzy set Ã in R is convex iff for any x_1, x_2 ε X , the membership function of Ã satisfies the inequality $\mu_{\tilde{A}}\{\lambda x_1+(1-\lambda)x_2\} \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$;

$0 \leq \lambda \leq 1$. Where min denotes the minimum operator

Definition

Normal fuzzy set : A fuzzy set Ã of the universal of discourse X is called a normal fuzzy set implying that there exists at least one x in X such that $\mu_{\tilde{A}}(x) = 1$

Definition

Trapezoidal Fuzzy number; A trapezoidal fuzzy number ã is denoted by (a_1,a_2, a_3, a_4) where a_1,a_2, a_3, a_4 are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given by :

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x \leq a_1, \\ \dfrac{x-a_1}{a_2-a_1}, & a_2 \leq x \leq a_1, \\ 1, & a_2 \leq x \leq a_3, \\ \dfrac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4, \\ 0, & x \geq a_4 \end{cases}$$

$\mu_{\tilde{a}}(x)$ satisfies the following conditions.

1. $\mu_{\tilde{a}}(x)$ is a continuous mapping from R to closed interval [0,1]
2. $\mu_{\tilde{a}}(x)=0$ for every xε(-∞,a_1]
3. $\mu_{\tilde{a}}(x)$ is strictly increasing and continuous [a_1,a_2]
4. $\mu_{\tilde{a}}(x)=1$ for every xε [a_2,a_3]
5. $\mu_{\tilde{a}}(x)$ is strictly decreasing and continuous on [a_3,a_4]
6. $\mu_{\tilde{a}}(x) = 0$ for every x ε [a_4,∞)

Definition

Triangular Fuzzy Number

A Triangular fuzzy number is denoted by (a_1,a_2, a_3,) Where a_1,a_2, a_3, are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given below:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ 1\ , & x = a_2, \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \end{cases}$$

Satisfying the following conditions

1. $\mu_{\tilde{a}}(x)$ is continuous mapping from R to closed interval [0,1]
2. $\mu_{\tilde{a}}(x)=0$ for every xε(-∞,a_1]
3. $\mu_{\tilde{a}}(x)$ is strictly increasing and continuous on [a_1,a_2]
4. $\mu_{\tilde{a}}(x)=1$ for x = a_2
5. $\mu_{\tilde{a}}(x)$ is strictly decreasing and continuous on [a_3,a_2]
6. $\mu_{\tilde{a}}(x) = 0$ for every x ε [a_3, ∞)

Definition

The α - cut set of a fuzzy set Ã is a crisp set defined by $\tilde{A}_\alpha = \{x \varepsilon X \mid \mu_{\tilde{A}}(x) \geq \alpha\}$

Arithmetic Operation

1. let $\tilde{A}$ = (a_1,a_2, a_3,) And $\tilde{B}$ = (b_1,b_2, b_3) be triangular fuzzy numbers then

   $\tilde{A} + \tilde{B}$ =(a_1,a_2, a_3,) + (b_1,b_2, b_3)
   = ($a_1 + b_1$, $a_2 + b_2$, $a_3 + b_3$)

2. Similarly, let $\tilde{A}$ = (a_1, a_2, a_3, a_4) and $\tilde{B}$ = (b_1, b_2, b_3, b_4) be trapezoidal fuzzy numbers then

   $\tilde{A} + \tilde{B}$ = (a_1,a_2, a_3, a_4) + (b_1, b_2, b_3, b_4)
   = ($a_1 + b_1$, $a_2 + b_2$, $a_3 + b_3$, $a_4 + b_4$)

Formulation of Replacement Model

Replacement policy can be classified into the following categories.

Case-a: When the equipment deteriorates with time and the value of money

1. does not change with time.
2. changes with time.

Case-b

When the units fail completely all of a sudden.

In this paper, we consider the case when the value of money does not change with time.

Our purpose is to determine the optimum replacement time of an item whose running or maintenance cost increases with time and the value of money remains static during the period:

Let

C : Capital cost of equipment.

S : Scrap value of equipment.

n : number of years that equipment would be in use.

f_t : maintenance cost for time t.

A(n) : Average total cost.

Here, we consider the case when, t is a discrete variable. If the equipment is used for n-years, then the cost incurred during this period is given by Total cost = Capital cost - Scrap value + Maintenance cost

$$T(c) = C - S + \sum_{t=1}^{n} f_t \text{ and its average cost is}$$

$$A(n) = \frac{C-S}{n} + \frac{1}{n}\sum_{t=1}^{n} f_t$$

Here A (n) will be minimum for the value of n, if $A(n+1) \geq A(n) \leq A(n-1)$.

Now, $A(n+1) = \frac{C-S}{n+1} + \frac{1}{n+1}\sum_{t=1}^{n+1} f_t$

$= \frac{1}{n+1}[C - S + \sum_{t=1}^{n} f_t + f_{(n+1)}]$

$= \frac{1}{n+1}[n A(n) + f_{(n+1)}]$

Therefore, $A(n+1) - A(n) = \frac{1}{n+1}[f_{(n+1)} - A(n)]$

Therefore, $A(n+1) \geq A(n) \Rightarrow f_{(n+1)} \geq A(n)$.

Similarly, $A(n) \leq A(n-1) \Rightarrow f_n \leq A(n-1)$.

So the replacement policy can be described as flows:

Replace the equipment at the end of n-th year, if the maintenance cost in the (n+1)th year is more than the average total cost in the nth year and the nth year maintenance cost is less than the previous year's average total cost.

Now, we investigate more realistic model of replacement problem with fuzzy capital cost ($\tilde{C}$) of equipment, fuzzy scrap value ($\tilde{S}$) of equipment and fuzzy maintenance cost ($\tilde{f}_t$). Since the objective is to determine that time, at which the replacement is most economical instead of continuing at an increase cost. So maintenance cost is also considered as TrFN or TFN.

The Proposed Model

The replacement problem when the value of money does not change with time and t is discrete variable. we rewrite the average annual cost as:

$$A(n) = \frac{C-S}{n} + \frac{1}{n}\sum_{t=1}^{n} f_t$$

Now, if capital cost ($\tilde{C}$), scrap value ($\tilde{S}$) and the maintenance cost ($\tilde{f}_t$) are fuzzy numbers, then the cumulative maintenance cost, total cost and average annual cost also fuzzy numbers.

So

$$\tilde{A}(n) = \frac{\tilde{C}-\tilde{S}}{n} + \frac{1}{n}\sum_{t=1}^{n} \tilde{f}_t \quad ....(1).$$

Hence, it cannot be determined the required time directly at which the replacement is economical instead of continuing at an increased cost. For solving this problem, we first transform capital cost, scrap value, and maintenance fuzzy cost as well as its corresponding average cost into a crisp one by a Yager's ranking method. It is a robust ranking technique, which satisfies linearity and additive properties and provides result, which is consistent with human intuition. Give a convex fuzzy member $\tilde{C}$, the Yager's Ranking index is defined by

Therefore, $Y(\tilde{C}) = \int_0^1 0.5(C_\alpha^L + C_\alpha^U)\,d\alpha$ (Where C_α^L, C_α^U) is a α-level cut of fuzzy number $\tilde{C}$.

The Yager's ranking index $Y(\tilde{C})$ presents the representative value of the fuzzy number ($\tilde{C}$)

If $\tilde{X} = a\tilde{W} + b\tilde{Z}$ and $\tilde{P} = g\tilde{Q} - h\tilde{R}$ where

$\tilde{X}, \tilde{W}, \tilde{Z}, \tilde{P}, \tilde{Q}$ and $\tilde{R}$ fuzzy numbers and a, b, g, h are constants.

Then we have,

$Y(\tilde{X}) = aY(\tilde{W}) + bY(\tilde{Z})$ and

$Y(\tilde{P}) = gY(\tilde{Q}) - hY(\tilde{R})$

Based on the properties the fuzzy replacement problem can be transformed in to crisp replacement problem.

if $Y(\tilde{U}) \leq Y(\tilde{V})$, then, using Yager's ranking method, we have $(\tilde{U}) \leq (\tilde{V})$ i.e. $\min\{\tilde{U}, \tilde{V}\} = \tilde{U}$.

Using linearity and additive properties of Yager's ranking method in (1) we get,

$$Y(\tilde{A}(n)) = Y\left(\frac{\tilde{C}-\tilde{S}}{n}\right) + \frac{1}{n}\sum_{t=1}^{n} Y(\tilde{f}_t) \quad (2)$$

$$Y(\tilde{A}(n)) = \frac{1}{n}Y(\tilde{C}-\tilde{S}) + \frac{1}{n}\sum_{t=1}^{n} Y(\tilde{f}_t) \quad (3)$$

$$Y(\tilde{A}(n)) = \frac{1}{n}[\{Y(\tilde{C}) - Y(\tilde{S})\} + \sum_{t=1}^{n} Y(\tilde{f}_t)] \quad (4)$$

where, $\tilde{A}$ (n) is the fuzzy average cost and $\tilde{f}_t$ is the fuzzy maintenance cost. Since $Y(\tilde{A}(n))$, $Y(\tilde{C})$, $Y(\tilde{S})$, $Y(\tilde{f}_t)$ gives a crisp value. So it can be solved by any conventional method.

Example

A construction company launched a certain type of loader whose fuzzy cost ($\tilde{C}$) in rupees in (61000, 61300, 61700, 62000) and the scrap value ($\tilde{S}$) is (4200, 4250, 4300, 4350). After survey, it has been noticed by company officials that the running cost (Maintenance Cost) in rupees are found from experience (see Table 1)

Table -1 : Yearly Maintenance Fuzzy Cost.

Year (n)	Maintenance fuzzy cost ($\tilde{f}$ t) of loader
1	$\tilde{f}$ 1 = [1200, 1350, 1400, 1450]
2	$\tilde{f}$ 2 = [2500, 600, 2750, 2900]
3	$\tilde{f}$ 3 = [3500, 3700, 3850, 4000]
4	$\tilde{f}$ 4 =[4500, 4650, 4800, 5000]
5	$\tilde{f}$ 5 = [6000, 6500, 6700, 6800]
6	$\tilde{f}$ 6 = [8000, 8200, 8450, 8800]
7	$\tilde{f}$ 7 = [10500, 11000, 12500, 14000]
8	$\tilde{f}$ 8 = [16000, 17000, 18500, 20000]

When would the company officials recommend replacing the loader with a new one ? (Take 1000=1 unit).

Solution :-(1) To solve this problem, we take Rs 1000=1 unit. Fist we find out the Yager's ranking index of all fuzzy cost i.e. maintenance cost ($\tilde{f}_t$), capital cost ($\tilde{C}$), and scrap value ($\tilde{S}$). So we get the following membership function and their corresponding indices.

$\tilde{f}_1$=(1.200, 1.350, 1.400, 1.450).

Therefore, we have

$$\mu_{f_t} = \begin{cases} 0, & x \le 1.2, \\ \frac{x-1.2}{1.35-1.2}, & 1.2 \le x \le 1.35 \\ =1, & 1.35 \le x \le 1.4 \\ \frac{1.45-x}{0.05}, & 1.4 \le x \le 1.45 \\ 0, & x \ge 1.45 \end{cases}$$

Therefore, α -cut of the fuzzy number (1.20, 1.35, 1.40, 1.45) is

$$(f_{1\alpha}^L, f_{1\alpha}^U) = (0.15\alpha + 1.2, 1.45 - 0.05\alpha)$$

Therefore, $Y(\tilde{f}_t)$ = Y(1.20,1.35,1.40,1.45) =

$$\int_0^1 0.5(f_{1\alpha}^L + f_{1\alpha}^U)\, d\alpha$$

$$= \int_0^1 0.5(0.15\alpha + 1.2 + 1.45 - 0.05\alpha)\, d\alpha$$

$$= \int_0^1 0.5(2.65 + 0.10\alpha)\, d\alpha$$

$$= \int_0^1 (1.325 + 0.05\alpha)\, d\alpha$$

= 1.325 + 0.025 = 1.3500

Similarly, we find the value of $Y(\tilde{f}_t)$ for t = 2, 3, , ...8, (see Table 2)

Table 2: Yager's ranking Index of Maintenance Fuzzy Cost

Year (n)	Yager's Ranking index of maintenance fuzzy cost ($\tilde{f}_t$) of loader
1	Y($\tilde{f}_1$) = 1.3500
2	Y($\tilde{f}_2$) = 2.6125
3	Y($\tilde{f}_3$) = 3.7625
4	Y($\tilde{f}_4$) = 4.7375
5	Y($\tilde{f}_5$) = 6.5000
6	Y($\tilde{f}_6$) = 8.3625
7	Y($\tilde{f}_7$) = 12.0000
8	Y($\tilde{f}_8$) = 17.8750

We also find out $Y(\tilde{C})$, $Y(\tilde{S})$ and their corresponding indices are 61.5000 and 4.2750 respectively. In order to determine the optimal time n, when the machine should be replaced, we calculate an average total cost per year during the life of the equipment. It is shown in the Table 3.

Table 3
Calculations to Determine Economic Life of Item

Year (n) (1)	Running Cost f(n) (2)	Cumulative Running Cost $\sum Y(\tilde{f}_t)$ (3)	Depreciation Cost Y $(\tilde{C})$ -Y$(\tilde{S})$ (4)	Total Cost (5)=(3)+(4) (5)	Average Cost (6)= $\frac{5}{n}$ (6)
1	1.3500	1.3500	57.225	58.5750	58.3500
2	2.6125	3.9625	57.225	61.1875	30.5937
3	3.7625	7.7250	57.225	64.9500	21.6500
4	4.7375	12.4625	57.225	69.6875	17.4218
5	6.5000	18.9625	57.225	76.1875	15.2375
6	8.3625	27.3250	57.225	84.5500	14.0916
7	12.0000	39.3250	57.225	96.5500	13.7928
8	17.8750	57.2000	57.225	114.450	14.3062

From the Table 3, it is clear that the minimum average annual cost is at the end of 7th year i.e. Rs 13792.8 (taking Rs 1000=1 unit). So, we can conclude that the loader should be replaced at the end of 7th year.

Example

A truck owner purchased three similar trucks with a fuzzy cost $(\tilde{C})$ = (8100000, 8150000, 8200000) in Rupees. He finds from his past records that the resale value of trucks is $(\tilde{S})$ = (202000, 204000, 206000) and maintenance fuzzy costs per hundred hours of these trucks are given in Table 4. (Take 100 hours = 1unit and Rs 1000=1 unit.) Determine at which time it is profitable to replace the truck with a new one?

Table 4. Maintenance Fuzzy Cost Per 100 Hours

Time(n)	**Maintenance fuzzy cost ($\tilde{f}_t$) of trucks.**
1	$\tilde{f}_1$ = [130, 140, 145]
2	$\tilde{f}_2$ = [250, 255, 260]
3	$\tilde{f}_3$ = [300, 315, 320]
4	$\tilde{f}_4$ = [375, 385, 390]
5	$\tilde{f}_5$ = [450, 465, 480]
6	$\tilde{f}_6$ = [600, 615, 630]
7	$\tilde{f}_7$ = [900, 925, 950]
8	$\tilde{f}_8$ = [1100, 1250, 1400]
9	$\tilde{f}_9$ = [1700, 1850, 1950]
10	$\tilde{f}_{10}$ = [2200, 2300, 2375]

Solution

Before solving this problem, we take 100 hours = 1 unit and Rs 1000 =1 unit. As per previous example, we first transform the FRP to a crisp replacement problem by Yager's Ranking index. So using this we have the Table 5.

Table 5 Yager's ranking index of maintenance fuzzy cost.

Time(n)	**Yager's Ranking Index of Maintenance fuzzy cost ($\tilde{f}_t$) of equipment.**
1	Y ($\tilde{f}_1$) = 138.75
2	Y ($\tilde{f}_2$) = 255.00
3	Y ($\tilde{f}_3$) = 312.50
4	Y ($\tilde{f}_4$) = 383.75
5	Y ($\tilde{f}_5$) = 465.00
6	Y ($\tilde{f}_6$) = 615.00
7	Y ($\tilde{f}_7$) = 925.00
8	Y ($\tilde{f}_8$) = 1250.00
9	Y ($\tilde{f}_9$) = 1837.00
10	Y ($\tilde{f}_{10}$) = 2293.75

Similarly, we find out Y $(\tilde{C})$, Y$(\tilde{S})$ and its corresponding values are 8150.00 and 204.00 respectively.

The average total cost per hundred hours is given in Table 6.

Table 6 : Calculation to determine economic life of item.

Hour (n) 1 unit = 100 (1)	Running Cost f(n) (2)	Cumulative Running Cost $\sum Y(\tilde{f_t})$ (3)	Depreciation Cost $Y(\tilde{C})$ - $Y(\tilde{S})$ (4)	Total Cost (5) = (3)+(4) (5)	Average Cost (6) = $\frac{f}{n}$ (6)
1	138.75	138.75	7946	8084.75	8084.75
2	255.00	393.75	7946	8339.75	4169.87
3	312.50	706.25	7946	8652.25	2884.08
4	383.75	1090.00	7946	9036.00	2259.00
5	465.00	1555.00	7946	9501.00	1900.20
6	615.00	2170.00	7946	10116.00	1686.00
7	925.00	3095.00	7946	11041.00	1577.28
8	1250.00	4345.00	7946	12291.00	1536.37
9	1837.00	6182.00	7946	14128.00	1569.77
10	2293.75	8475.00	7946	16421.00	1642.10

From Table 6, we see that the average cost per hundred hours is minimum, at the end of 800 hours (taking 1unit=100 hours) i.e., Rs 1536370 (Taking 1unit=Rs1000). Therefore, we conclude that the truck owner should expect to replacement a truck to a new one at the end of 800 hours.

Conclusion

In this paper, the fuzzy replacement problem is considered in the sense that the capital cost, scrap value, maintenance or running cost are all imprecise in nature represented by fuzzy numbers. It is more realistic and closer to our daily life situation. Here, we use Yager's ranking method to deal with this type of FRP. This method shifts the FRP to crisp RP with its corresponding ranking indices. This method is simple and easy to apply for solving practical FRP. Two numerical examples have been provided to show the simplicity and effectiveness of the proposed method. It also shows that proper decision for replacement of FRP can be made easily. We hope that the proposed method may be used for future study in FRP when the value of money dependent on time.

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